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CHAPTER-1 CLASS-9 REAL NUMBERS.

Types of Numbers :

We have learnt about number 1, 2, 3, ...
These are called Natural Numbers denoted by N.

If we include 0. They are called
Whole Numbers denoted by symbol W.

If we include the negative of these numbers.
These all these number are called Integers.
denoted by symbol Z.

Are there still any numbers left on the number line ?
Yes these are fractions. such as $\frac{5}{7}$ etc.

and we can see that all these above numbers can be either Natural or whole or Integers can be written in these form. Therefore these are called Rational numbers. So, it is a big set of Numbers. R

So, we can say that there are infinidely many rational numbers between two rational numbers.

Q. find 6 rational numbers between $\frac{3}{7}$ and $\frac{5}{7}$.

~~Suppose~~ $\cancel{\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{6}{14} \; \frac{9}{21}}$

$$\cancel{\frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21}}$$

$$\frac{3}{7} = \frac{3 \times 4}{7 \times 4} = \frac{12}{28}$$

$$\frac{5}{7} = \frac{5 \times 4}{7 \times 4} = \frac{20}{28}.$$

So, 6 rational numbers are $\frac{13}{28}, \frac{14}{28}, \frac{15}{28}, \frac{16}{28}, \frac{17}{28}, \frac{18}{28}$.

Rational Numbers:-

All numbers in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. Such numbers are called Rational numbers.

Ex :- $\frac{5}{7}$, $\frac{-2}{8}$, $\frac{399}{788}$.

Irrational Numbers :

But there are still some numbers which cannot be written in the form $\frac{p}{q}$, where p and q are integers.

Ex :- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π etc.

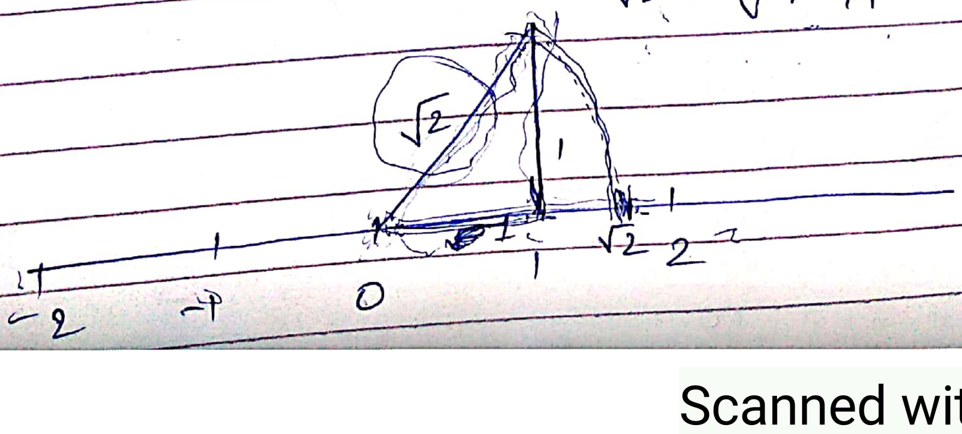
Proof of irrationality of these numbers will be done in class 10.

So, these numbers complete the set of all the numbers which can be drawn on a number line. All these numbers are called Real Numbers denoted by symbol R.

Representation of Irrational Numbers on Number line.

1) Represent $\sqrt{2}$ on a number line.

$$\sqrt{2} = \sqrt{1^2+1^2}$$



2. Polynomials.

A Polynomial is a special type of algebraic Expression, which has only whole number in the exponents of the variable of the polynomial.

Ex :- $x^2 + 2x + 7$. Examples which aren't polynomials are $x + \frac{1}{x}$, $\sqrt{x} + 3$. etc.

Polynomial in One Variable.

If a polynomial consist of only one variable, then it is called Polynomial in One variable.

Ex :- $x + 1$, $x^2 + 2x + 2$, $y^3 + y$ etc.

~~Degree of a Polynomial~~

Terms and Coefficient of a polynomial

Terms → The algebraic expressions separated by '+' sign in an algebraic expression are called Terms.

Ex :- In the polynomial $3x^2 + 2x + 2$. $3x^2$, $2x$ and 2 are the three terms.

Coefficients :→ The Constants in each term of a polynomial are called Coefficients of those terms.

Ex :- In polynomial $3x^2 + 2x - 2$, x^2 has a coefficient 3 and x has a coefficient 2.

Degree of a polynomial :-

The highest power of the variable of a polynomial is called the Degree of the polynomial. For Ex —

The polynomial $2x+3$ has a degree 1.
" " $2x^3-2x^2+3$ has a degree 3.

So,

1) A Polynomial of degree 0 is called a Constant Polynomial.
Ex :- 3, -5 etc.

2) A Polynomial of degree 1 is called a Linear Polynomial.
Ex :- $2x+3$, $x-2$ etc.

3) A Quadratic Polynomial is a polynomial of degree 2.
Ex :- $3x^2+2x-1$, $4x^2-1$ etc.

4) A cubic Polynomial is a polynomial of degree 3.
Ex :- x^3-2x^2+7x-1, x^3-2x etc.

A monomial is a polynomial with 1 term
A binomial is a polynomial with 2 terms.
A trinomial is a polynomial with 3 terms.

Exercise - 2.4

4. Factorise

(i) $12x^2-7x+1$

Soln:- $12x^2-7x+1 = 12\left(x^2 - \frac{7}{12}x + \frac{1}{12}\right)$

$= 12\,p(x)$

If a and b are the zeroes of $p(x)$

then, $12x^2-7x+1 = 12(x-a)(x-b)$.

So, $ab = \frac{1}{12}$.

So, the possible values of a and b are

$\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6},$

Now, $P\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \frac{7}{12}\left(\frac{1}{2}\right) + \frac{1}{12}$

$= \frac{1}{4} - \frac{7}{24} + \frac{1}{12} = \frac{6-7+2}{24} = \frac{1}{24} \neq 0.$

So, $\left(x-\frac{1}{2}\right)$ is not a factor of $P(x)$.

Then, $P\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 - \frac{7}{12}\left(-\frac{1}{2}\right) + \frac{1}{12}$

$= \frac{1}{4} + \frac{7}{24} + \frac{1}{12} = \frac{6+7+1}{24} \neq 0.$

$P\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^2 - \frac{7}{12}\left(\frac{1}{3}\right) + \frac{1}{12}$

$= \frac{1}{9} - \frac{7}{36} + \frac{1}{12} = \frac{4-7+3}{36} = \frac{7-7}{36} = 0.$

So, $\left(x-\frac{1}{3}\right)$ is a factor of $P(x)$

So, if $a = \frac{1}{3}$. Then, $b = \frac{1}{4}$.

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So, $p'(x) = \left(x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)$

So,

$12x^2 - 7x + 1 \; \cancel{p(x)} = 12\left(x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)$

$= \overset{3 \times 4}{\cancel{12}}\left(x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)$

~~So, $p(x) = (3x-1)(4x-1)$~~

$\cancel{p(x)} = (3x-1)\,(4x-1)$

(iii) $6x^2+5x-6$

Soln:- $6x^2+5x-6 = 6\left(x^2+\frac{5}{6}x-1\right)$

~~If~~ $= 6\,P(x)$

If a and b are the roots of $P(x)$. Then

$P(x) = (x-a)(x-b) = x^2+\frac{5}{6}x-1$

So, ~~Hence~~

So, $ab = -1$

and $a+b = \frac{5}{6}$.

So, the possible values of a and b. Can be $\pm\frac{2}{3}$ and $\pm\frac{3}{2}$.

So,

$P\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 + \frac{5}{6}\left(\frac{2}{3}\right) - 1$

$= \frac{4}{9} + \frac{10}{18} - 1$

$= \frac{8+10-18}{18} = 0.$

So, $\left(x-\frac{2}{3}\right)$ is a factor of $P(x)$.

and The other will be $-\frac{3}{2}$.

$P\left(-\frac{3}{2}\right) = \left(\frac{-3}{2}\right)^2 + \frac{5}{6}\left(\frac{-3}{2}\right) - 1$

$= \frac{9}{4} - \frac{15}{12} - 1$

$$p(x) = \left(x - \frac{2}{3}\right)\left(x + \frac{3}{2}\right)$$

and

$$6x^2 + 5x - 6 = 6\,p(x)$$

$$= 6\left(x - \frac{2}{3}\right)\left(x + \frac{3}{2}\right)$$

$$= 3 \times 2\left(x - \frac{2}{3}\right)\left(x + \frac{3}{2}\right)$$

So, $6x^2 + 5x - 6 = (3x - 2)(2x + 3)$.

5. Factorise.

(i) $x^3 - 2x^2 - x + 2$

Here ~~let~~ $P(x)$ is supposed to be $x^3 - 2x^2 - x + 2$.

So,

$$P(x) = x^3 - 2x^2 - x + 2$$

The factors of 2 are $\pm 1, \pm 2$.

So,

$$P(1) = (1)^3 - 2(1)^2 - 1 + 2$$
$$= 1 - 2 - 1 + 2 = 0.$$

So,

$(x-1)$ is a factor of $P(x)$

Now,

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline x-1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & -x^2 - x + 2 \\ & -x^2 + x \\ & + \quad - \\ \hline & -2x + 2 \\ & -2x + 2 \\ & + \quad - \\ \hline & 0. \end{array}$$

So, $P(x) = (x-1)(x^2 - x - 2)$

Now, we can factorise again $(x^2 - x - 2)$ by factor theorem,

If $g(x) = x^2 - x - 2 = (x-a)(x-b)$.

Then, $ab = -2$.

possible values of a & b are $\pm 1, \pm 2$.

$$g(1) = 1^2 - 1 - 2 \neq 0.$$

$$g(-1) = (-1)^2 - (-1) - 2 = 1 + 1 - 2 = 0.$$

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So, $a = -1$

and $b = 2$

So,

$x^3 - 2x^2 - x + 2 = (x-1)(x+1)(x-2)$.

(iv) $2y^3 + y^2 - 2y - 1$

(iv) $2y^3 + y^2 - 2y - 1$

Then, $p(y) = 2y^3 + y^2 - 2y - 1$

The factors of -2 are $\pm 1, \pm 2$.

So, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) - 2 + 1 + 2 - 1 = 0$.

So, $(y+1)$ is a factor of $p(y)$.

$$\begin{array}{r} 2y^2 - y - 1 \\ y+1 \overline{) 2y^3 + y^2 - 2y - 1} \\ \underline{\underset{-}{2y^3} \underset{-}{+} 2y^2} \\ -y^2 - 2y - 1 \\ \underline{\mp y^2 \mp y} \\ -y - 1 \\ \underline{-y - 1} \\ 0. \end{array}$$

So, $p(y) = (y+1)(2y^2 - y - 1)$

So, Now factorising $2y^2 - y - 1$

If $g(y) = 2y^2 - y - 1 = 2(y-a)(y-b)$.

Then, ~~possible values~~ $= 2(y^2 - \frac{1}{2}y - \frac{1}{2})$

So, $ab = -\frac{1}{2}$.

possible values of a and b are, $\pm 1, \pm \frac{1}{2}$.

So, $g(1) = 1^2 - \frac{1}{2} - \frac{1}{2}$

$= 1 - \frac{1}{2} - \frac{1}{2} = 0$.

So, $(y-1)$ is a factor of $g(y)$

So, if $a = 1$ Then, $b = -\frac{1}{2}$

So, $p(y) = (y+1)\,2(y-1)\left(y+\frac{1}{2}\right)$

$= (y+1)(y-1)(2y+1)$

Algebraic Identities :-

1. $(x+y)^2 = x^2 + 2xy + y^2$
2. $(x-y)^2 = x^2 - 2xy + y^2$.
3. $x^2 - y^2 = (x+y)(x-y)$
4. $(x+a)(x+b) = x^2 + (a+b)x + ab$.
5. $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$.
6. $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
7. $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
8. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$